W1 L10 - SOLVING LINEAR DIFFERENTIAL EQUATIONS WITH AN INTEGRATING FACTOR

EX

$$\frac{dy}{dx} + 3y = 2xe^{-3x} \qquad P(x) = 3$$

$$\mu(x) = e^{\int 3\,dx} \Rightarrow \mu(x) = e^{3x}$$

$$e^{3x} \cdot \frac{dy}{dx} + e^{3x} 3y = e^{3x} \cdot 2xe^{-3x}$$

$$e^{3x} \frac{dy}{dx} + 3e^{3x} y = 2x$$

$$\int D_x \left[e^{3x} \cdot y \right] dx = \int 2x\,dx$$

$$e^{3x} y = x^2 + C$$

$$y = (x^2 + C) e^{-3x}$$

EX

$$\frac{dy}{dx} - 2xy = e^{x^2} \qquad P(x) = -2x$$

$$\mu(x) = e^{\int -2x\,dx} \Rightarrow \mu(x) = e^{-x^2}$$

$$e^{-x^2} \frac{dy}{dx} - 2xy \cdot e^{-x^2} = e^{x^2} e^{-x^2}$$

$$e^{-x^2} \frac{dy}{dx} - 2xye^{-x^2} = 1$$

ALWAYS GOING TO BE $D_x[\mu(x) \cdot y]$

$$\int D_x \left[e^{-x^2} \cdot y \right] dx = \int 1\,dx$$

$$e^{-x^2} \cdot y = x + C$$

$$y = (x + C) e^{x^2}$$

EX

$$x\frac{dy}{dx} + 5y = 7x^2 \quad ; \quad y(2) = 5$$

$$\frac{dy}{dx} + \frac{5}{x}\cdot y = 7x \qquad P(x) = 5/x$$

$$\mu(x) = e^{\int 5/x\,dx} \Rightarrow \mu(x) = e^{5\ln|x|} \Rightarrow e^{\ln x^5} \Rightarrow \mu(x) = x^5$$

$$x^5\frac{dy}{dx} + x^5\cdot\frac{5}{x}y = 7x\cdot x^5$$

$$x^5\frac{dy}{dx} + 5x^4 y = 7x^6$$

$$\int D_x\left[x^5\cdot y\right]dx = \int 7x^6\,dx$$

$$x^5\cdot y = x^7 + C$$

$$2^5\cdot 5 = 2^7 + C \Rightarrow C = 32$$

$$x^5 y = x^7 + 32$$

$$y = x^2 + 32/x^5$$